Who Was Fibonacci?

~ Born in Pisa, Italy in 1175 AD

~ Full name was Leonardo Pisano

~ Grew up with a North African education under the Moors

~ Traveled extensively around the Mediterranean coast

~ Met with many merchants and learned their systems of arithmetic

~ Realized the advantages of the Hindu-Arabic system

Fibonacci's Mathematical Contributions

~ Introduced the Hindu-Arabic number system into Europe

~ Based on ten digits and a decimal point

~ Europe previously used the Roman number system

~ Consisted of Roman numerals

~ Persuaded mathematicians to use the Hindu-Arabic number system

1 2 3 4 5 6 7 8 9 0 and .

I = 1

V = 5

X = 10

L = 50

C = 100

D = 500

M = 1000

Fibonacci’s Mathematical Contributions Continued

~ Wrote five mathematical works

~ Four books and one preserved letter

~ *Liber Abbaci* (*The Book of Calculating*) written in 1202

~ *Practica geometriae* (*Practical Geometry*) written in 1220

~ *Flos* written in 1225

~ *Liber quadratorum* (*The Book of Squares*) written in 1225

~ *A letter to Master Theodorus* written around 1225

The Fibonacci Numbers

~ Were introduced in *The Book of Calculating*

~ Series begins with 0 and 1

~ Next number is found by adding the last two numbers together

~ Number obtained is the next number in the series

~ Pattern is repeated over and over

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, …

$$\mathrm{F}n + 2 = \mathrm{F}n + 1 + \mathrm{F}n$$

Fibonacci's Rabbits

Problem:

Suppose a newly-born pair of rabbits (one male, one female) are put in a field. Rabbits are able to mate at the age of one month so that at the end of its second month, a female can produce another pair of rabbits. Suppose that the rabbits never die and that the female always produces one new pair (one male, one female) every month from the second month on. How many pairs will there be in one year?

Fibonacci's Rabbits Continued

~ End of the first month = 1 pair

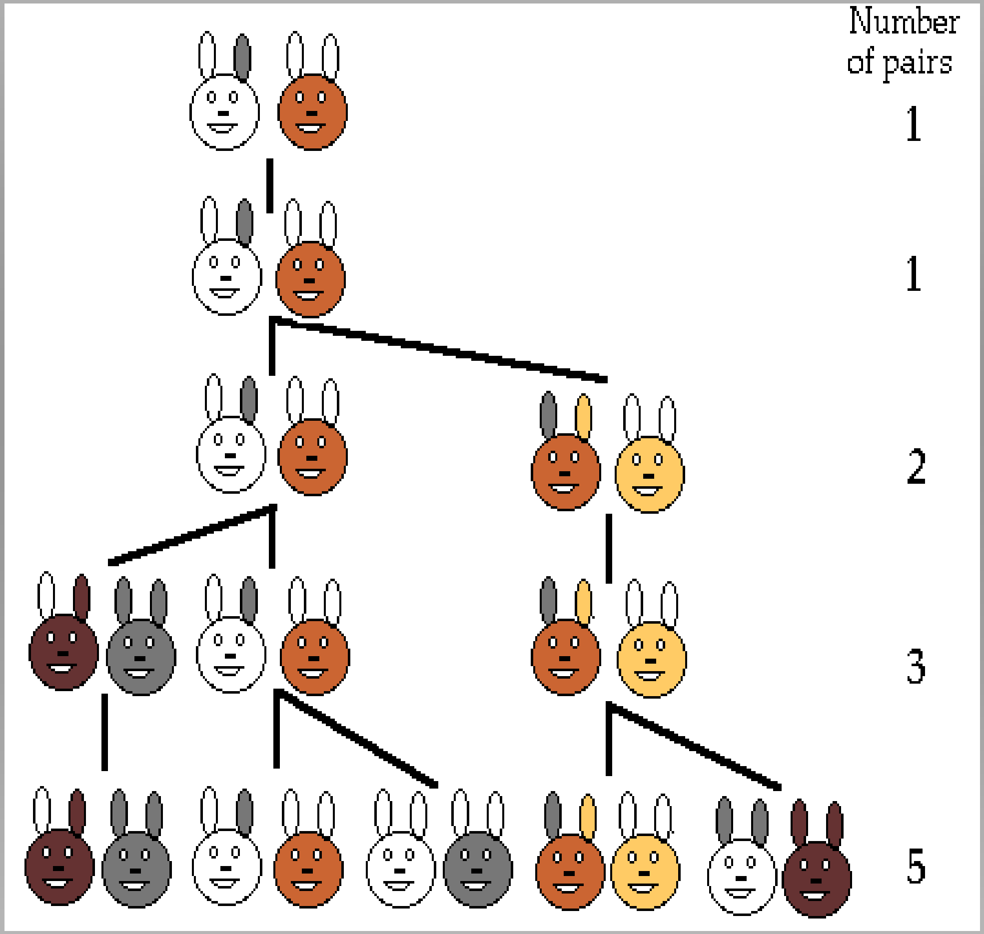
~ End of the second month = 2 pair

~ End of the third month = 3 pair

~ End of the fourth month = 5 pair

~ 5 pairs of rabbits produced in one year

1, 1, 2, 3, 5, 8, 13, 21, 34, …



The Fibonacci Numbers in Pascal's Triangle

~ Entry is sum of the two numbers either side of it, but in the row above

~ Diagonal sums in Pascal's Triangle are the Fibonacci numbers

~ Fibonacci numbers can also be found using a formula

```
        1
      1   1
    1   2   1
  1   3   3   1
1   4   6   4   1
```

1				
1	1			
1	2	1		
1	3	3	1	
1	4	6	4	1

$$\text{Fib}(n) = \sum_{k=1}^{n} \binom{n-k}{k-1}$$

The Fibonacci Numbers and Pythagorean Triangles

~ The Fibonacci numbers can be used to generate Pythagorean triangles

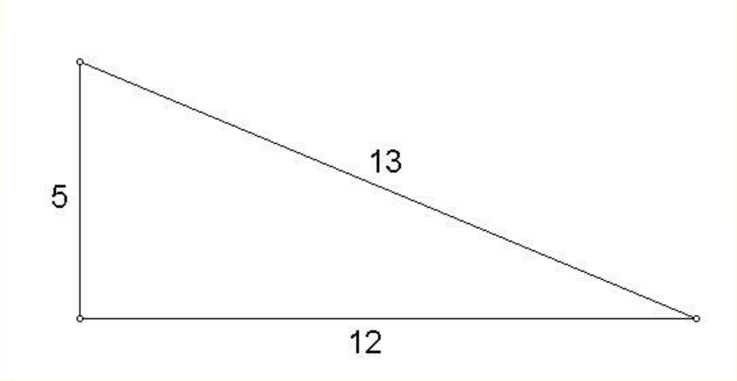
~ First side of Pythagorean triangle = 12

~ Second side of Pythagorean triangle = 5

~ Third side of Pythagorean triangle = 13

~ Fibonacci numbers 1, 2, 3, 5 produce Pythagorean triangle 5, 12, 13

a	b	$a + b$	$a + 2b$
1	2	3	5



The Fibonacci Numbers and Geometer's Sketchpad

Construction:

~ On any line segment AB construct any rectangle ABCD

~ Find midpoints P2 of segment CD, M1 of segment AD, and M2 of segment BC

~ Draw segment M1M2

~ Draw segment AP2

~ Draw diagonal BD

~ Let S be the point of intersection of segment AP2 and segment BD

~ The foot of the altitude from S to segment CD is P3, where DP = (1/3)DC

~ Let S' be the point of reflection of S over segment M1M2

~ Draw segment S'D

~ Draw segment AP3

~ Let K be the point of intersection of segment AP3 and segment S'D

~ The foot of altitude from K to segment CD is P5, where DP5 = (1/5)DC

~ Let K' be the point of reflection of K over segment M1M2

~ Draw segment K'D

~ Draw segment AP5

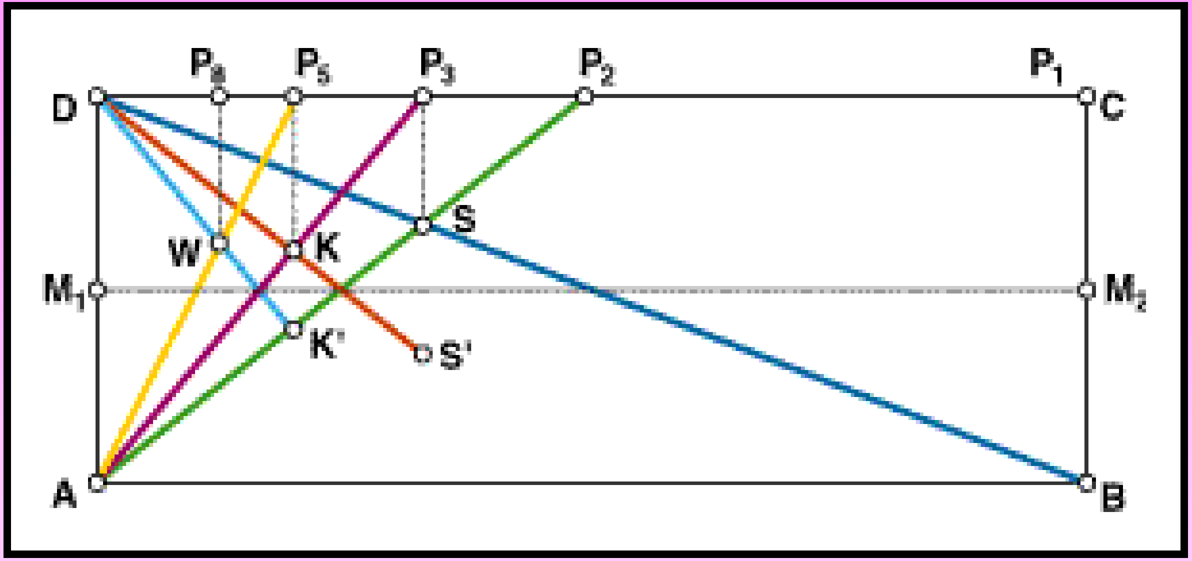
~ Let W be the point of intersection of segment AP5 and segment K'D

~ The foot of the altitude from W to segment CD is P8, where DP8 = (1/8)CD

The Fibonacci Numbers and Geometer's Sketchpad Continued

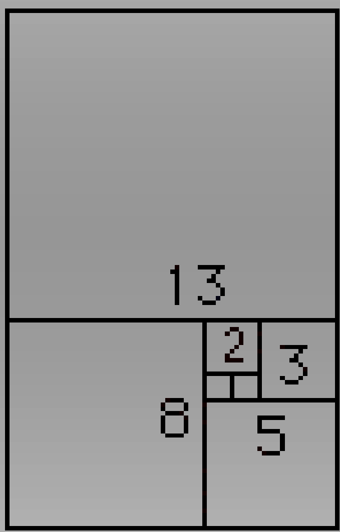
~ All points in the pattern are reciprocals of the Fibonacci numbers

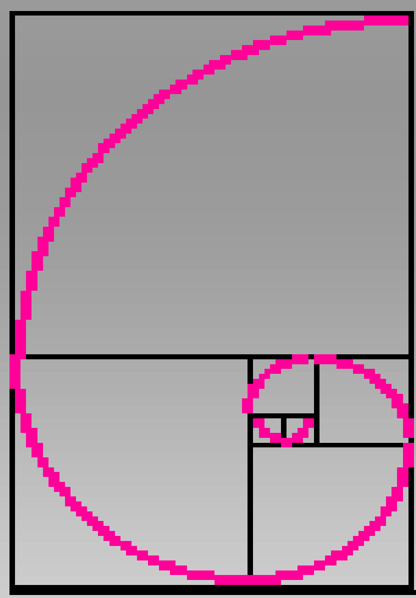
1/1, 1/2, 1/3, 1/5, 1/8, 1/13, 1/21, 1/34, …

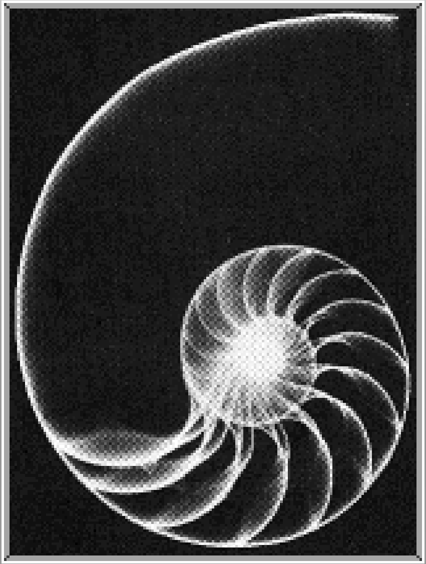


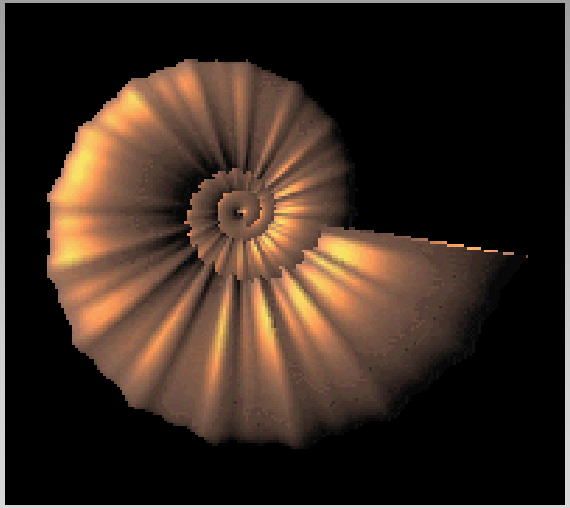
The Fibonacci Numbers in Nature

~ Fibonacci spiral found in both snail and sea shells



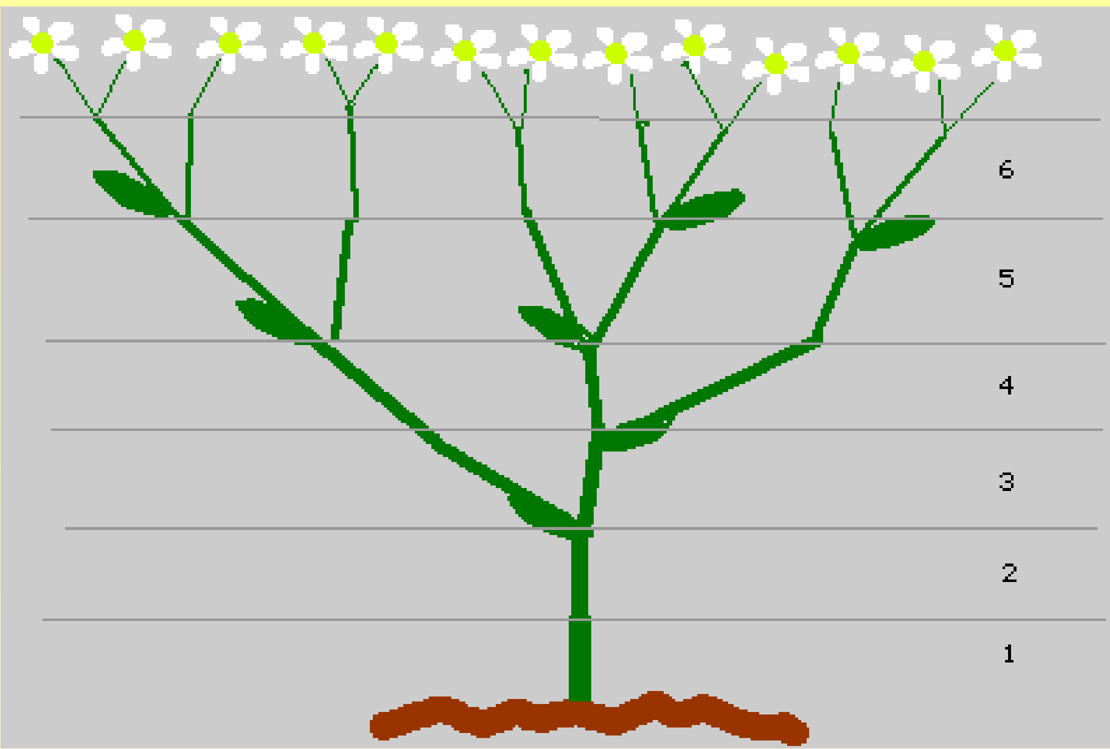






The Fibonacci Numbers in Nature Continued

~ Sneezewort (Achillea ptarmica) shows the Fibonacci numbers

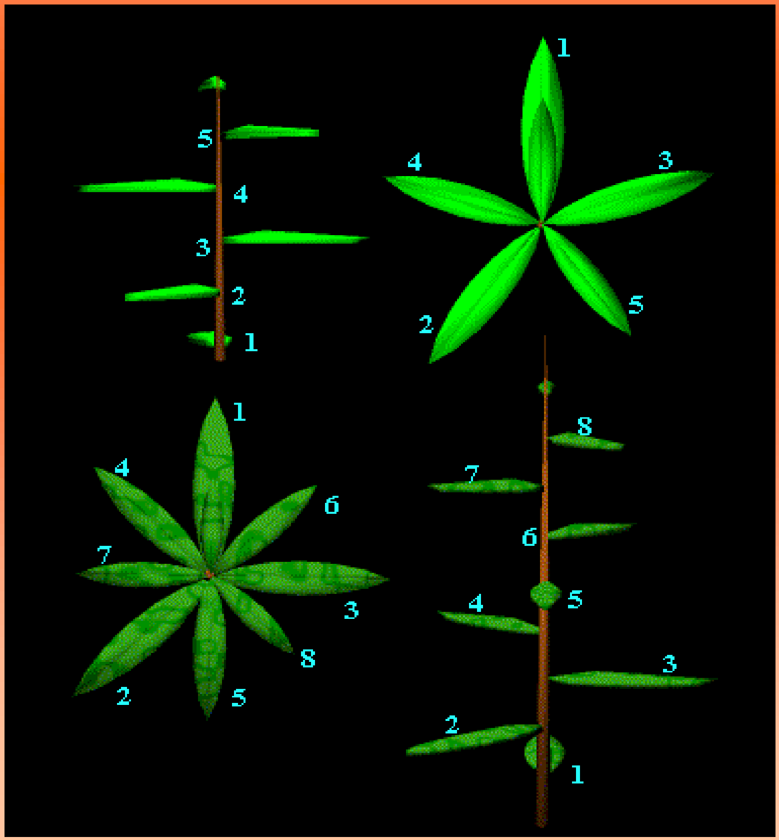


The Fibonacci Numbers in Nature Continued

~ Plants show the Fibonacci numbers in the arrangements of their leaves

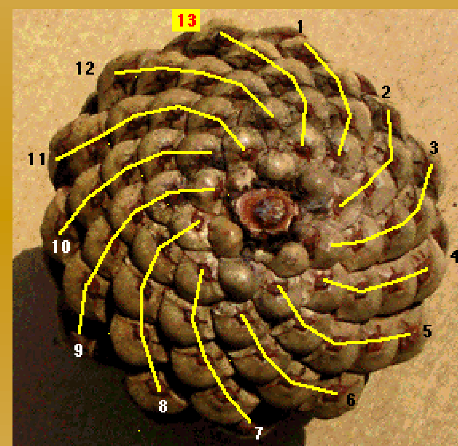
~ Three clockwise rotations, passing five leaves

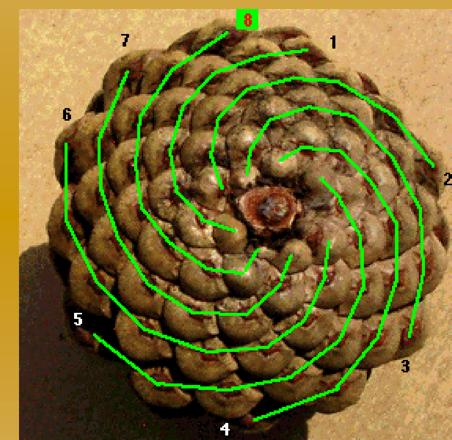
~ Two counter-clockwise rotations

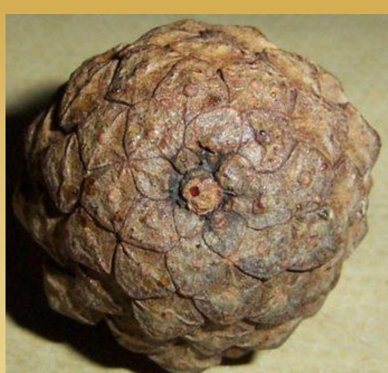


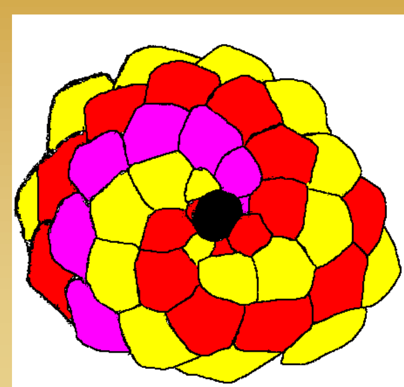
The Fibonacci Numbers in Nature Continued

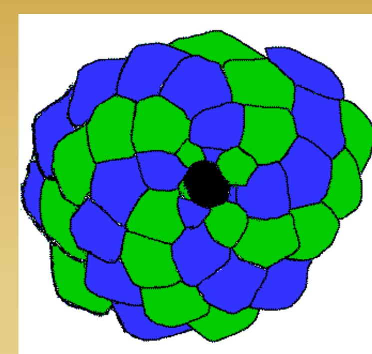
~ Pinecones clearly show the Fibonacci spiral











The Fibonacci Numbers in Nature Continued


Lilies and irises = 3 petals


Buttercups and wild roses = 5 petals

Corn marigolds = 13 petals

Black-eyed Susan’s = 21 petals

The Fibonacci Numbers in Nature Continued

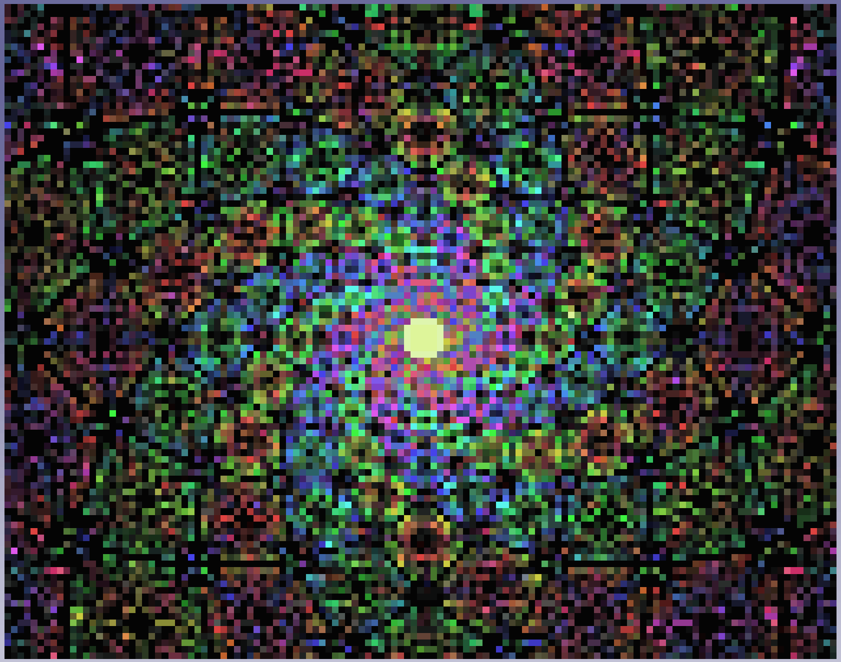
~ The Fibonacci numbers are found in the arrangement of seeds on flower heads

~ 55 spirals spiraling outwards and 34 spirals spiraling inwards

The Fibonacci Numbers in Nature Continued

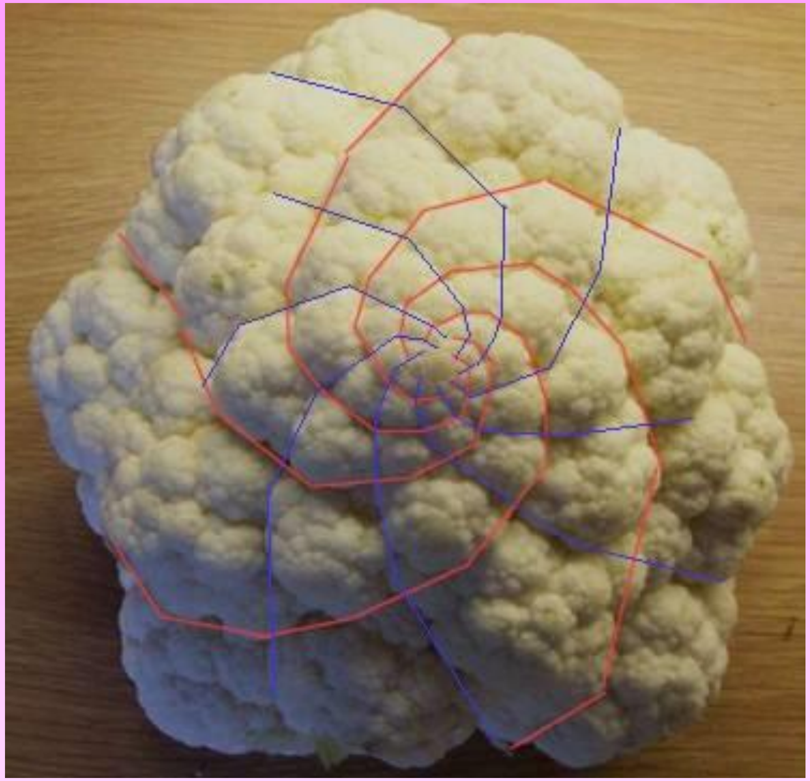
~ Fibonacci spirals can be made through the use of visual computer programs

~ Each sequence of layers is a certain linear combination of previous ones



The Fibonacci Numbers in Nature Continued

~ Fibonacci spiral can be found in cauliflower



The Fibonacci Numbers in Nature Continued

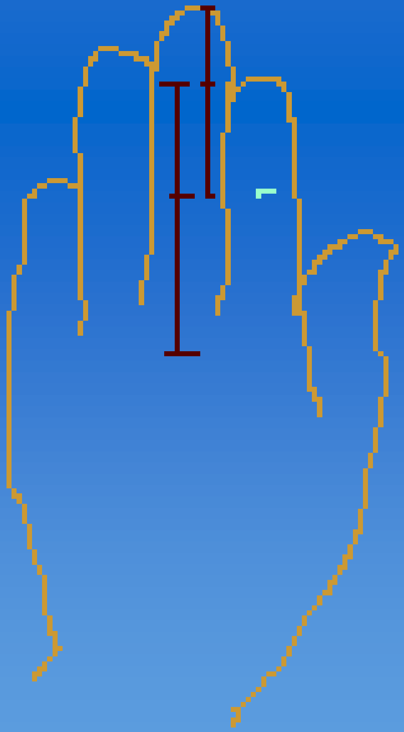
~ The Fibonacci numbers can be found in pineapples and bananas
~ Bananas have 3 or 5 flat sides
~ Pineapple scales have Fibonacci spirals in sets of 8, 13, 21

The Fibonacci Numbers in Nature Continued

~ The Fibonacci numbers can be found in the human hand and fingers

~ Person has 2 hands, which contain 5 fingers

~ Each finger has 3 parts separated by 2 knuckles



The Golden Section

~ Represented by the Greek letter Phi

~ Phi equals ± 1.6180339887 … and ± 0.6180339887 …

~ Ratio of Phi is 1 : 1.618 or 0.618 : 1

~ Mathematical definition is $\text{Phi}^2 = \text{Phi} + 1$

~ Euclid showed how to find the golden section of a line

< - - - - - - - - 1 - - - - - - - - >

A G B

g $1 - g$

$$\frac{GB}{AG} = \frac{AG}{AB} \quad \text{or} \quad \frac{1 - g}{g} = \frac{g}{1}$$

so that $g^2 = 1 - g$

The Golden Section and The Fibonacci Numbers

~ The Fibonacci numbers arise from the golden section

~ The graph shows a line whose gradient is Phi

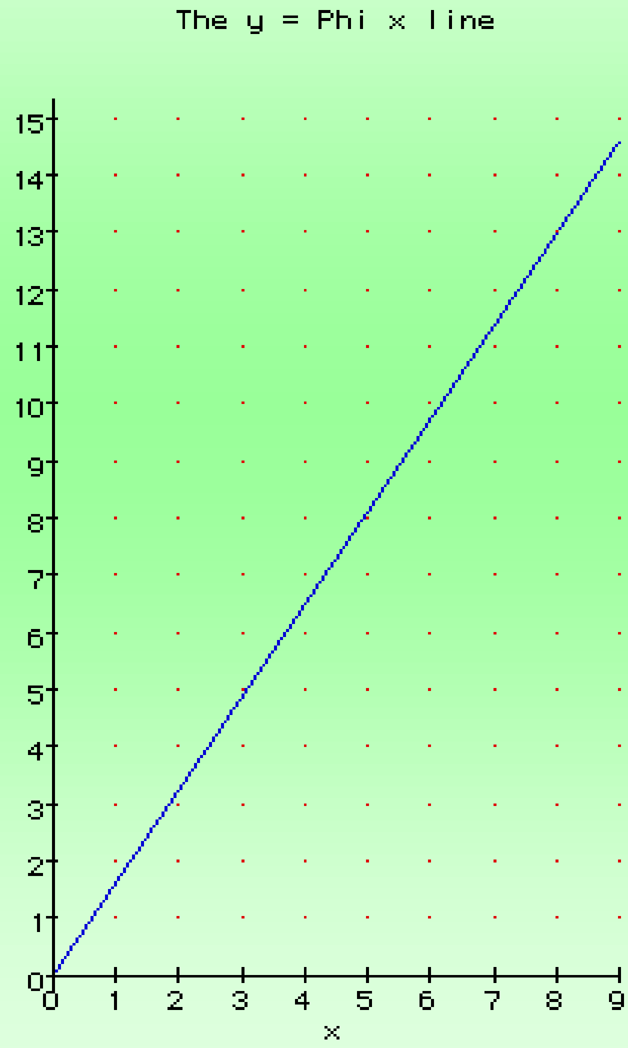
~ First point close to the line is (0, 1)

~ Second point close to the line is (1, 2)

~ Third point close to the line is (2, 3)

~ Fourth point close to the line is (3, 5)

~ The coordinates are successive Fibonacci numbers



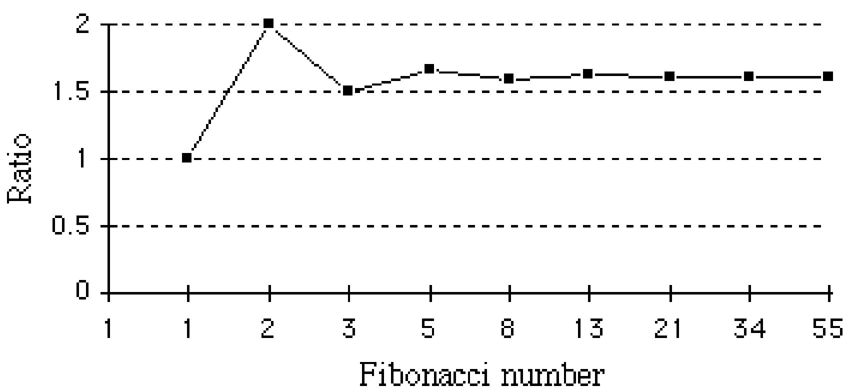
The Golden Section and The Fibonacci Numbers Continued

~ The golden section arises from the Fibonacci numbers

~ Obtained by taking the ratio of successive terms in the Fibonacci series

~ Limit is the positive root of a quadratic equation and is called the golden section

$$\frac{1}{1}, \frac{2}{1}, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}, \ldots$$



If you take two successive terms of the series, a, b, and $a+b$ then

$$\frac{b}{a} \cong \frac{a+b}{b}$$

$$\cong \frac{a}{b} + 1$$

We define the golden section, ϕ (*phi*), to be the limit of $\frac{b}{a}$, so:

$$\phi = \frac{1}{\phi} + 1$$

$$\phi^2 - \phi - 1 = 0$$

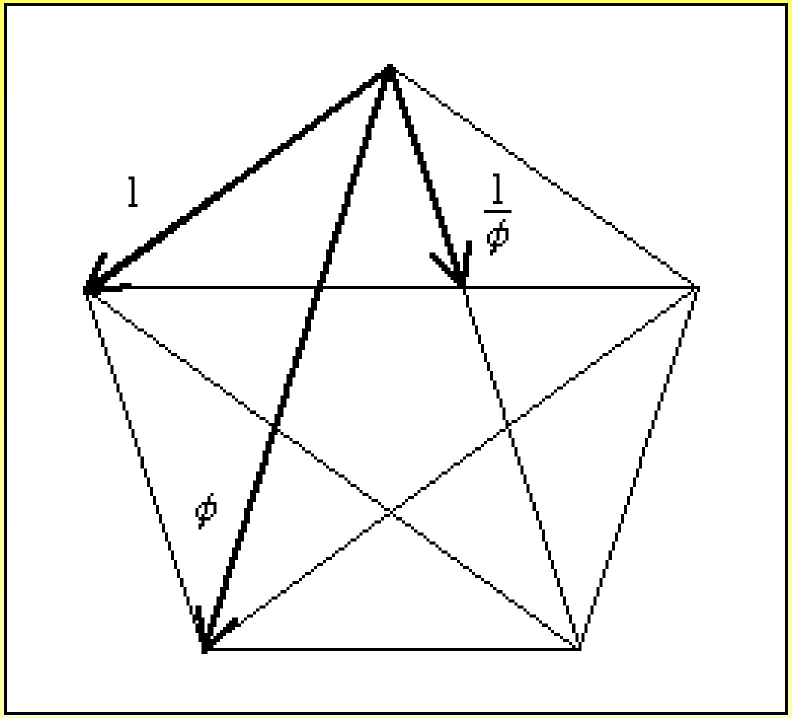
$$\phi = \frac{1+\sqrt{5}}{2} \approx 1.618$$

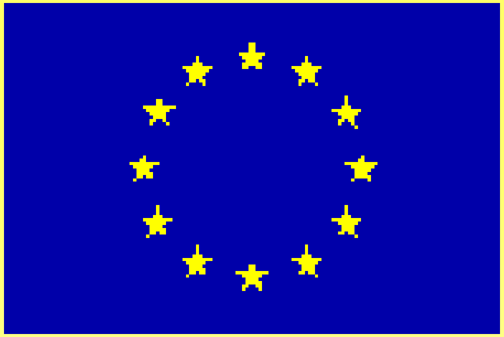
The Golden Section and Geometry

~ Is the ratio of the side of a regular pentagon to its diagonal

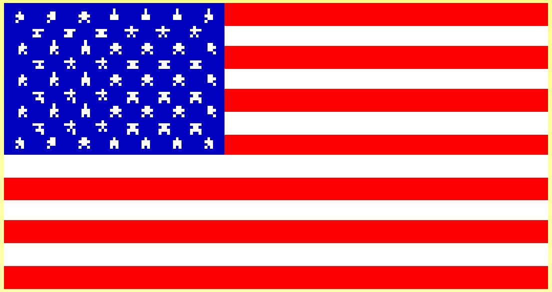
~ The diagonals cut each other with the golden ratio

~ Pentagram describes a star which forms parts of many flags





European Union



United States

The Golden Section in Nature

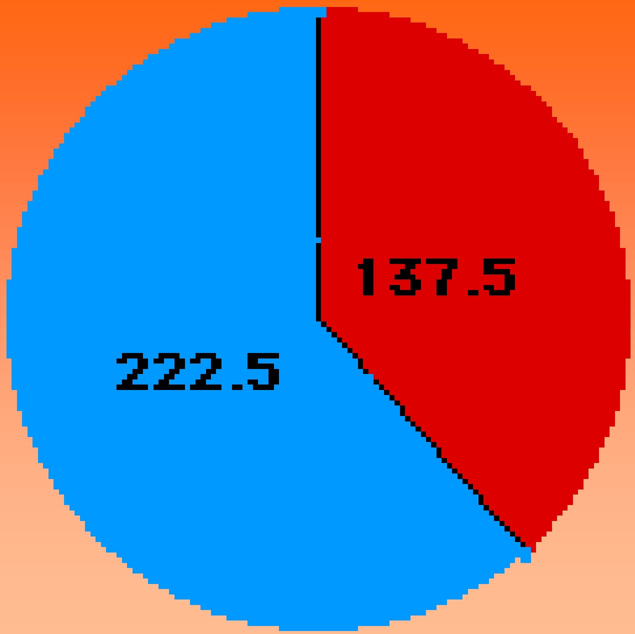
~ Arrangements of leaves are the same as for seeds and petals

~ All are placed at 0.618 per turn

~ Is 0.618 of 360^o which is 222.5^o

~ One sees the smaller angle of 137.5^o

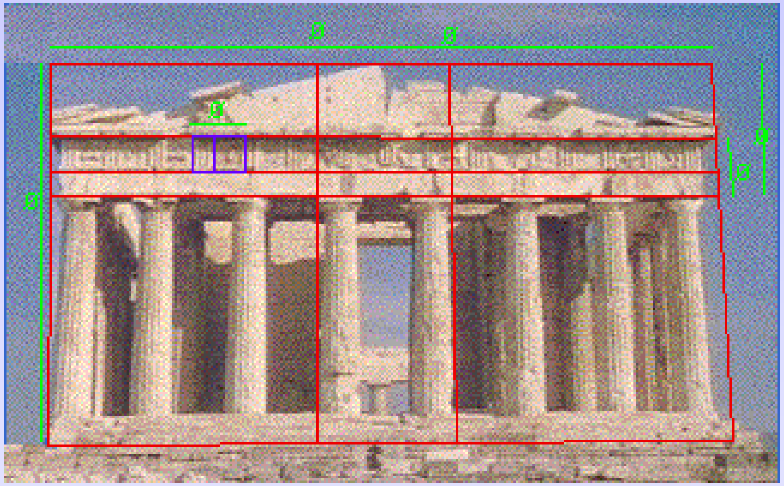
~ Plants seem to produce their leaves, petals, and seeds based upon the golden section

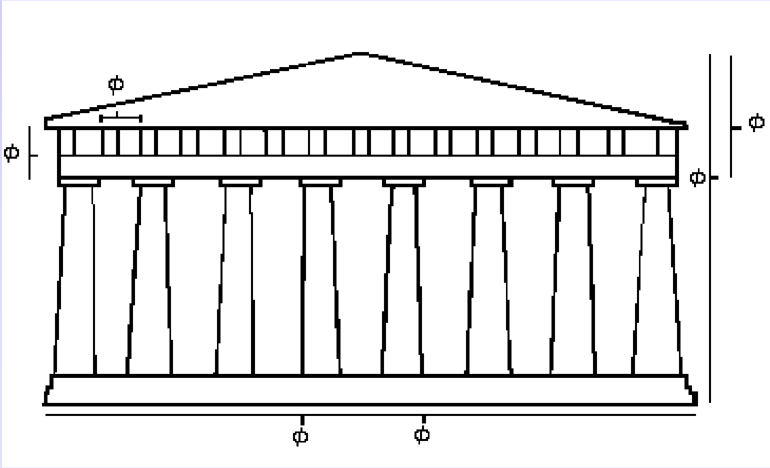


The Golden Section in Architecture

~ Golden section appears in many of the proportions of the Parthenon in Greece

~ Front elevation is built on the golden section (0.618 times as wide as it is tall)

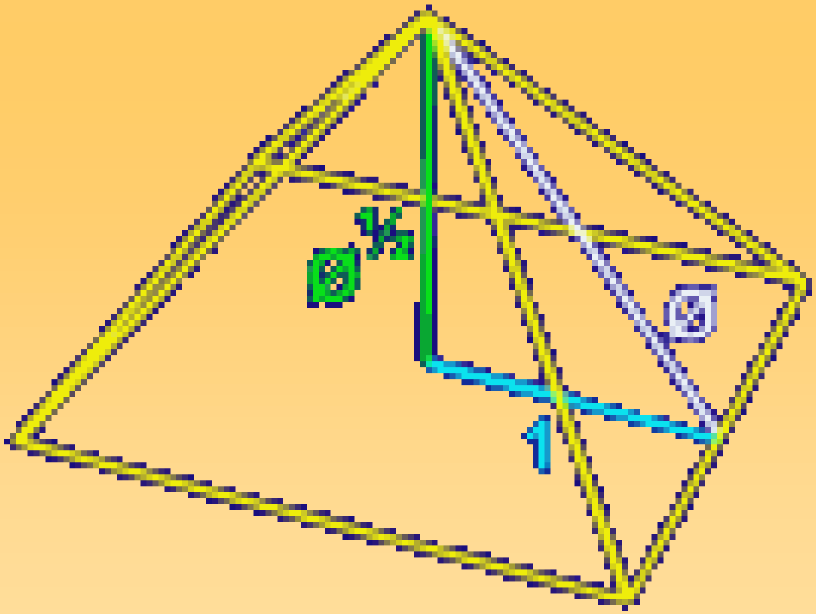




The Golden Section in Architecture Continued

~ Golden section can be found in the Great pyramid in Egypt

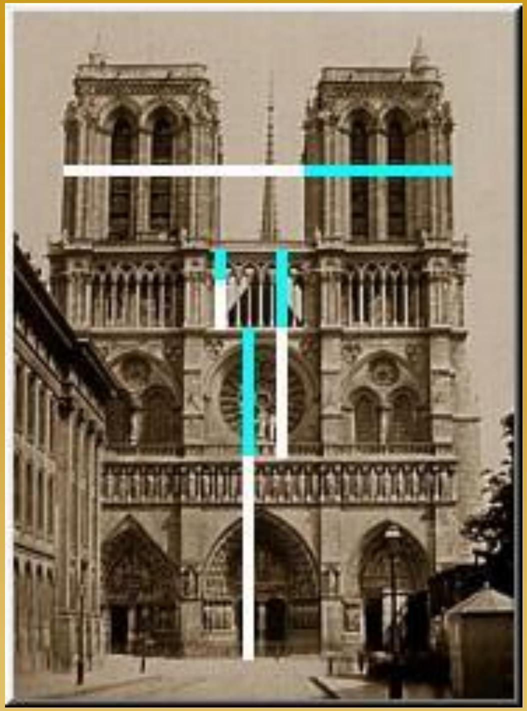
~ Perimeter of the pyramid, divided by twice its vertical height is the value of Phi



The Golden Section in Architecture Continued

~ Golden section can be found in the design of Notre Dame in Paris

~ Golden section continues to be used today in modern architecture



United Nations Headquarters

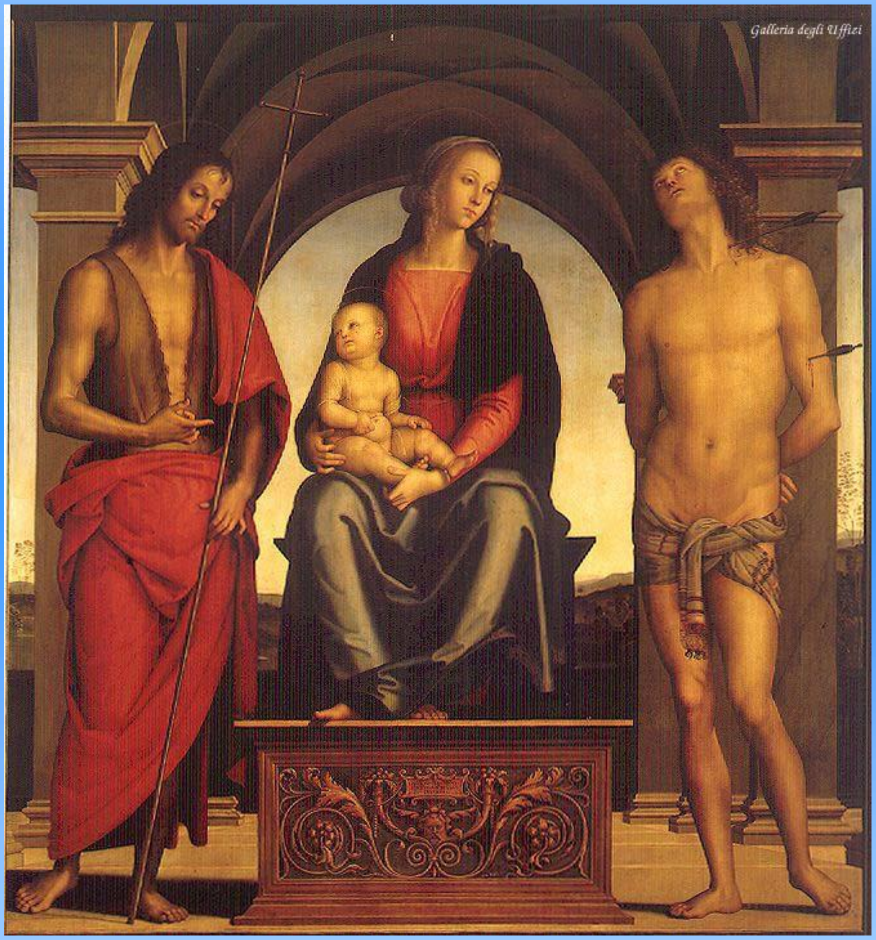
Secretariat building

The Golden Section in Art

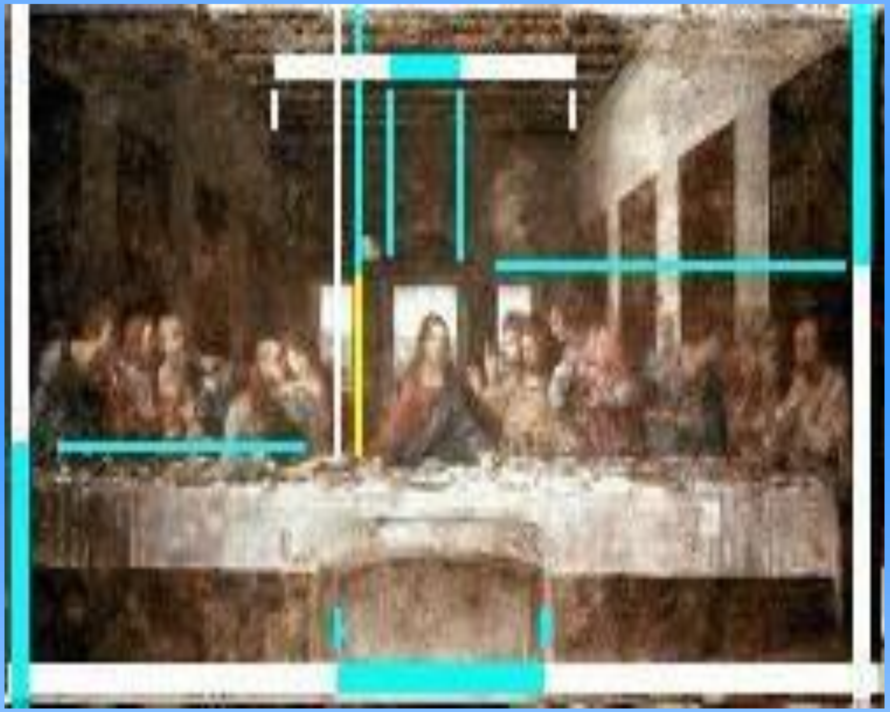
~ Golden section can be found in Leonardo da Vinci's artwork

The Annunciation

The Golden Section in Art Continued



Madonna with Child and Saints



The Last Supper

The Golden Section in Art Continued

~ Golden section can be seen in Albrecht Durer’s paintings

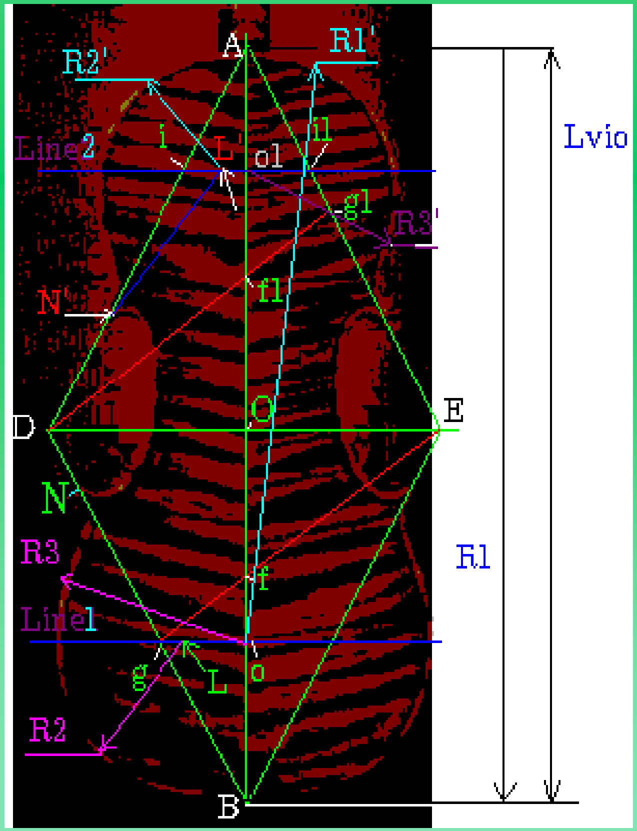
Trento

Nurnberg

The Golden Section in Music

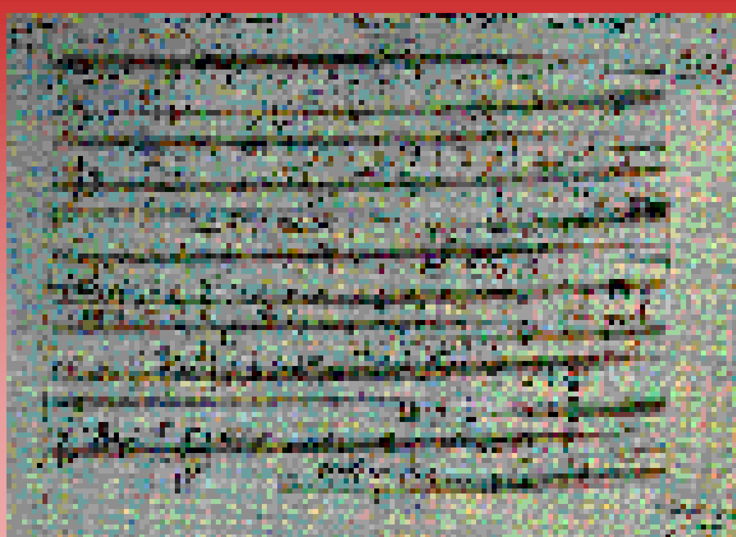
~ Stradivari used the golden section to place the f-holes in his famous violins

~ Baginsky used the golden section to construct the contour and arch of violins

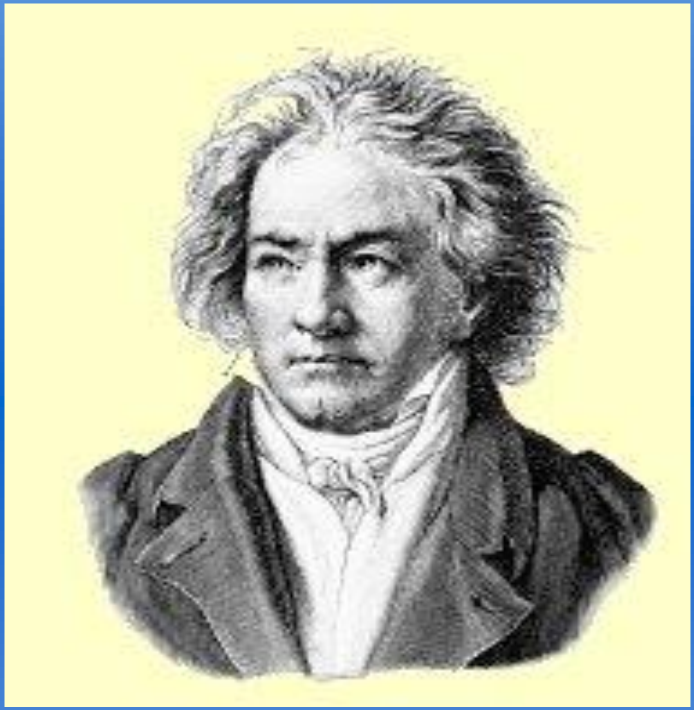


The Golden Section in Music Continued

- ~ Mozart used the golden section when composing music
- ~ Divided sonatas according to the golden section
- ~ Exposition consisted of 38 measures
- ~ Development and recapitulation consisted of 62 measures
- ~ Is a perfect division according to the golden section



The Golden Section in Music Continued



~ Beethoven used the golden section in his famous *Fifth Symphony*

~ Opening of the piece appears at the golden section point (0.618)

~ Also appears at the recapitulation, which is Phi of the way through the piece

The Fibonacci Numbers and The Golden Section at Towson

The Fibonacci Numbers and The Golden Section at Towson Continued

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